14.8 Lagrange Multipliers

Another way to find max/min.

Entry Task (*motivation*)

Consider $z = f(x, y) = 2x + 3y$.

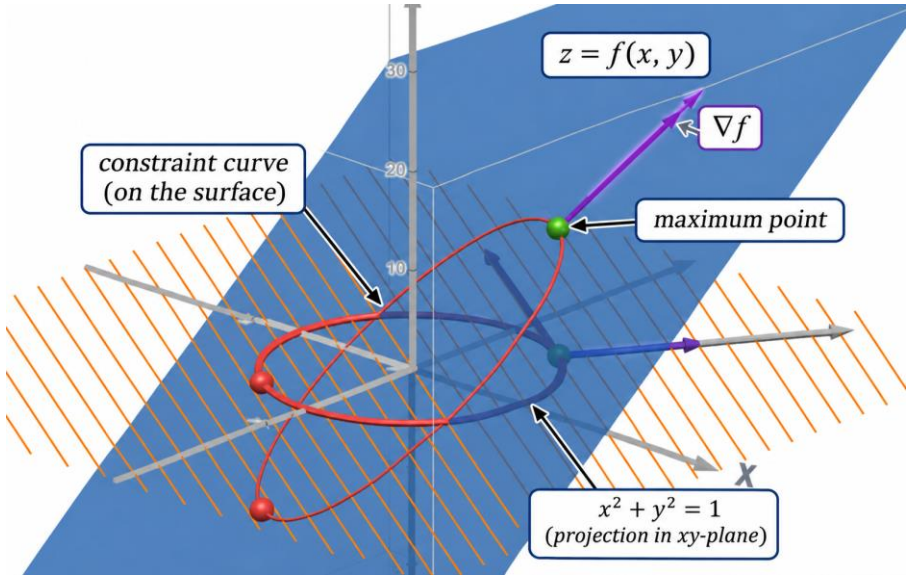
1. Draw the level curves for $z = k$ with k = 0, 2, and 4.
2. Now draw $x^2 + y^2 = 1$.
3. Label on circle the point at which you think $f(x, y)$ is largest.

Visual: https://www.desmos.com/3d/nj96whntnn

Lagrange Multipliers

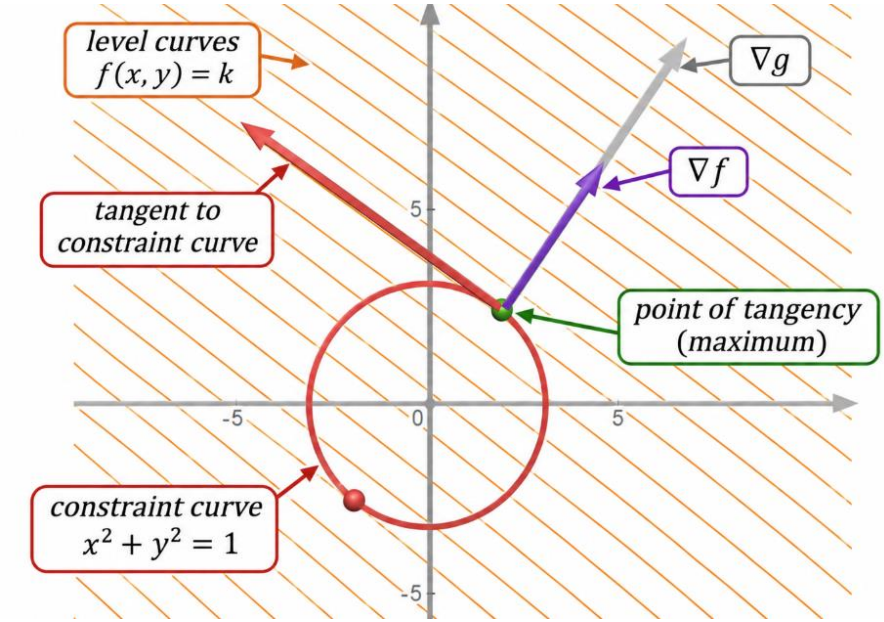
At a maximum or minimum of differentiable function $f(x, y)$ subject to $g(x, y) = c$, either:

1. The gradients ∇f and ∇g are parallel, or
2. The point is an endpoint (*or a point where $\nabla g = 0$ or DNE)*



If ∇f and ∇g are parallel, then for come constant λ ,

$$\nabla \mathrm{f} = \lambda \nabla \mathrm{g}.$$



The Method

To find the maximum and minimum of $f(x, y)$ subject to $g(x, y) = c$.

Set up

- $f_x = \lambda g_x$,
- $f_y = \lambda g_y$,
- $g(x, y) = c$.

Solve the system for candidate points (x, y).

Evaluate $f(x, y)$ at each candidate and endpoint.

Compare.

Some Algebra to try

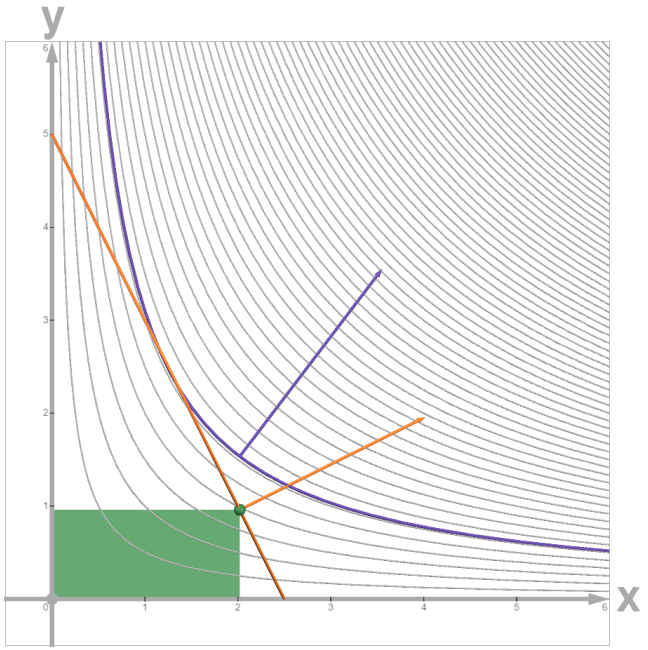
1. Solve for x, y in terms of λ, if straightforward, and combine.
2. Eliminate λ, which gives
$$f_x g_y - f_y g_x = 0$$
3. Solve together with the constraint.

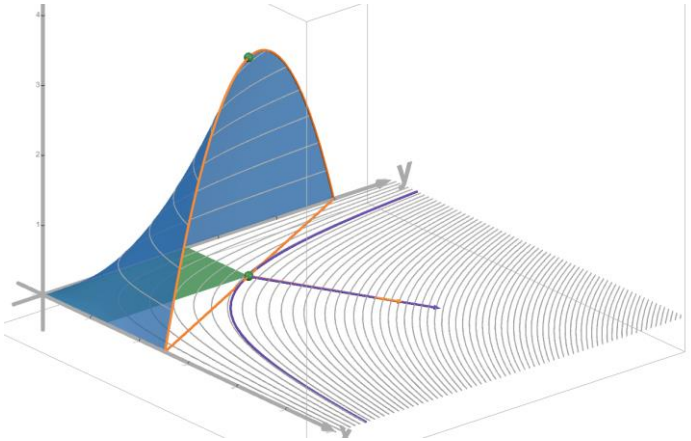
Example 1

Find the maximum of

$$f(x, y) = xy$$

subject to $y = 5 - 2x$ with $x \geq 0$ and $y \geq 0$





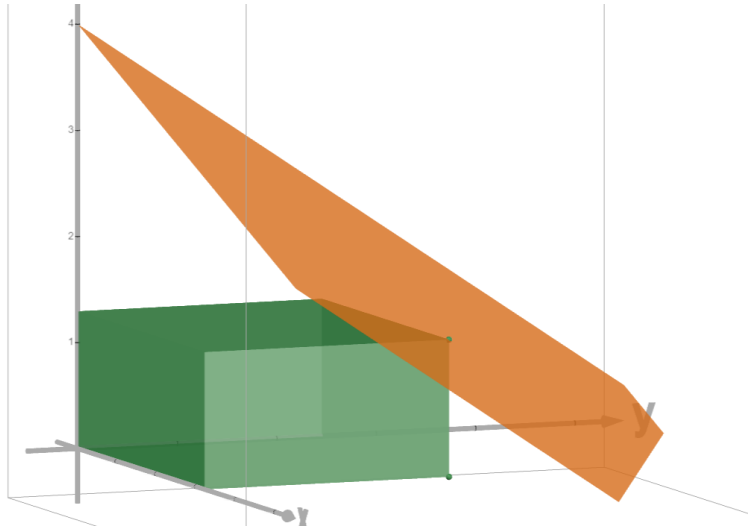
Application Visual: https://www.desmos.com/3d/ps3syzvfm9

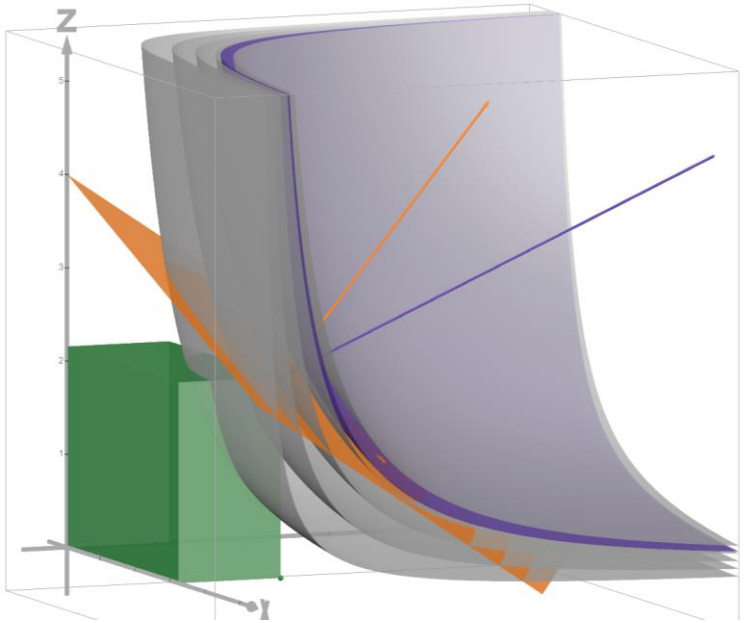
Example 2

Find the maximum of

$$f(x, y, z) = xyz$$

subject to $x + 2y + 3z = 12$ with $x > 0$, $y > 0$, and $z > 0$.

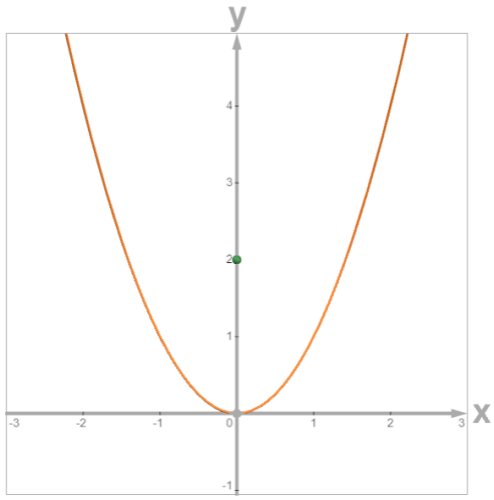


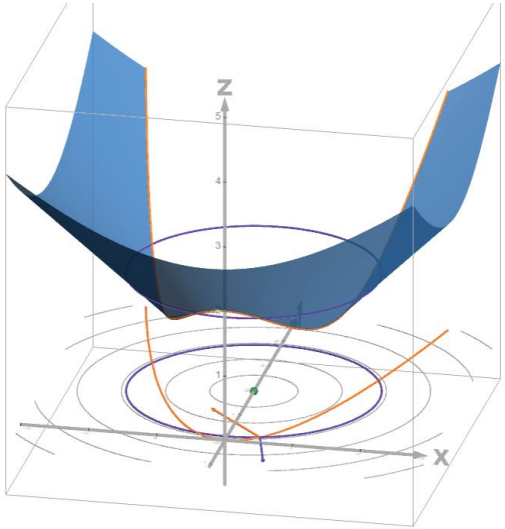


Application Visual: https://www.desmos.com/3d/7tt1qicup3

Example 3

Find the point(s) on $y = x^2$ that are closest to $(0,2)$.





Application Visual: https://www.desmos.com/3d/twcdyhwmtb

Example 4

Find the point on $x^2 + y^2 - z^2 = 4$

closest to $P = (3,4,0)$.

